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A PRELIMINARY STUDY OF THE ORBITS OF MERIT FOR MOON PROBES

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by
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SUMMARY

A preliminary study has been made of ideal space vehicle orbits for moon probes by the method of successive approximation carried out on the IBM 7090 digital computer, under the approximation implied in the restricted three-body problem. It has been found that orbits are possible on which the space vehicle can encircle both the earth and the moon at reasonable distances for a period of a few years or more. The practical procedure by which such orbits can be systematically generated solely by numerical methods is given here, and some examples are also given.

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INTRODUCTION

Ideal space vehicle orbits for moon probes pass closely around the moon and afterwards approach close to the earth so that whatever information the space vehicle has gathered around the moon can be transmitted back to the earth. We can devide such orbits into two groups: (1) orbits which enclose the earth and the moon and which pass both at short distances a number of times, and (2) orbits which first pass closely around the moon and then return to the same geographical point at which the vehicle was launched. Orbits of the second kind will not be considered here; this study will concentrate only on those orbits of the first kind which leads us to the problem of searching for orbits in the well known problem of three bodies in celestial mechanics. With the advent of high-speed electronic computers, the time has come to approach the three-body problem numerically. This is especially true for any problem such as the present one where the time scale involved is not astronomically long but rather is humanly short. In other words, the coming of astronautics has introduced into celestial mechanics many new problems of empirical nature which can be solved more easily in an unorthodox way by means of numerical experiments than in the standard manner by mathematical analysis.

Several papers (References 1, 2, and 3) have appeared recently in which the periodic orbits in the restricted three-body problem have been derived partly by means of numerical computations. In this report, the emphasis is on the practical procedure by which the orbits that may be used for actual moon probes can be systematically generated solely by numerical methods.

THE STARTING CONDITION OF NUMERICAL EXPERIMENTS

If the desirable orbits are to be obtained numerically (by successive approximation), there must be some starting conditions. As the starting condition we employ those orbits enclosing both the earth and the moon which have periods commersurable with the period of the moon and which pass relatively close to the earth as well as to the moon. The effect of the moon is neglected in the treatment of the starting condition; therefore, in this section only the orbits of the vehicle in the gravitational field of the earth alone are considered. In the next section, the perturbation by the moon, insofar as the motion of the vehicle can be approximated by the restricted three-body problem, will be considered in the process of successive approximation. Of course, once the desired orbits within the framework of the restricted three-body problem have been found, those actual ones in the earth-moon-sun system may be obtained by further successive approximation in the same manner as the desired orbits in the restricted three-body problem have been generated here from the starting condition based upon two-body approximation. However, this preliminary study does not go beyond the approximation implied in the restricted three-body problem.

Let the semimajor axes of the orbits of the moon and of the space vehicle around the earth be 1 and a and let their periods be P_0 and P, respectively. Figure 1 illustrates the orbit $S_1 S_2 S_3 \ldots S_7$ of the vehicle and the orbit $M_1 M_2 M_3 \ldots M_7$ of the moon with the earth at point E. We suppose the vehicle to enter orbit at S_1 when the moon is at the position M_1 on its orbit. If the two periods P_0 and P have a ratio given by

$$\frac{P}{P_0} = \frac{2m}{2n + 1} \tag{1}$$

where both m and n are integers, and if

$$(1 + e)\,a = \alpha \tag{2}$$

where e represents the eccentricity of the vehicle's orbit and α is a numerical factor of the order of unity such that $(\alpha - 1)$ measures the proximity of the vehicle's approach to the moon, the vehicle will repeatedly reach the moon and return to the neighborhood of the earth. The time interval between two consecutive encounters of the vehicle with the moon is $2m$ sidereal months if the perturbing effect of the moon is neglected. The actual time interval has to be computed by integrating the equations of motion of the vehicle.

If the space vehicle is launched at a point between the earth and the moon and on the line joining them, the ratio of P_0 and P would be

$$\frac{P}{P_0} = \frac{2m + 1}{2n + 1} \,. \tag{3}$$

In general,

$$\frac{P}{P_0} = \frac{2m'l + 2}{2n'l + l} \qquad \begin{cases} l = 1, 2, 3, \ldots\ldots \\ m' = 0, 1, 2, \ldots\ldots \\ n' = 0, 1, 2, \ldots\ldots \end{cases} \tag{4}$$

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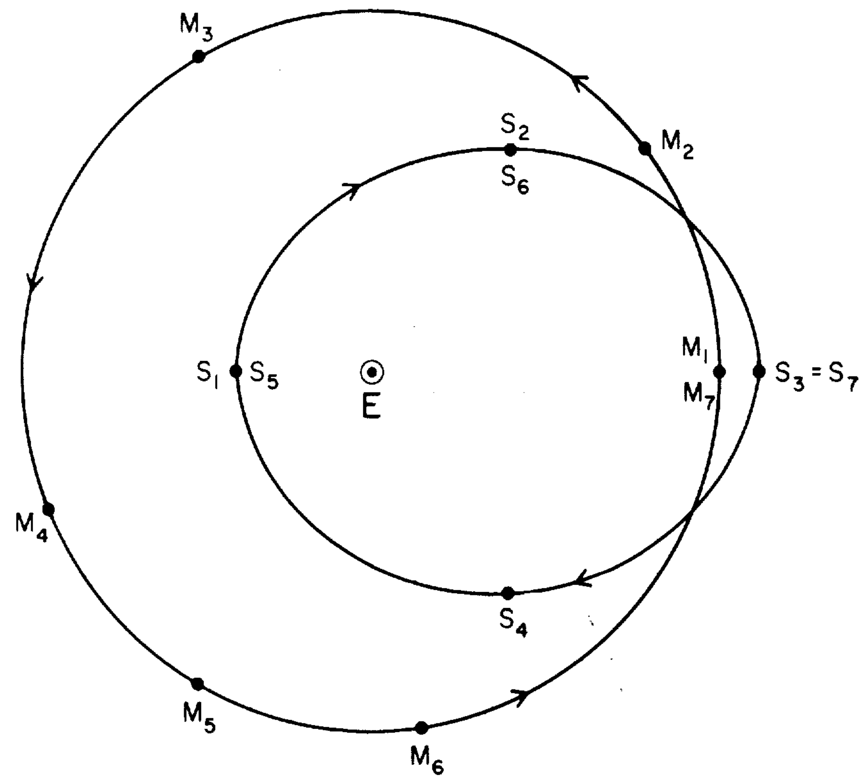

Figure 1 – An illustration of the periodic encounters between the moon and the space vehicle. The space vehicle is launched at S_1 when the moon is at the position M_1. When the moon moves around the earth E from M_1 to M_2 to ... to M_7, the space vehicle moves from S_1 to S_2 to ... to S_7. The moon's perturbing effect on the motion of the vehicle is neglected here.

It is obvious that the commeasurability of P and P_0 may be simply written as a ratio of two relative prime numbers. However, by writing the condition in the somewhat cumbersome form of Equation 4 we have the advantage of being able to see at once what position the moon should be in when the satellite is launched. Thus, Equation 4 reduces to Equation 1 for $l = 1$, $m' = m - 1$, $n' = n$, and reduces to Equation 2 for $l = 2$, $m' = m$, $n' = n$. Both Message (Reference 2) and Newton (Reference 3) have studied the case for $P/P_0 = 1/2$ corresponding to $l = 4$, $m' = n' = 0$. In this paper we shall consider the situations arising from Equation 1.

Equation 2 is based on the assumption that the encounter between the moon and the space vehicle occurs at the apogee of the space vehicle's orbit. If an encounter at perigee is desired, a minus sign should be used in place of the plus sign in Equation 2. However, from practical considerations, only the encounter at apogee is interesting. Consequently, this discussion will be limited to encounters at apogee.

It can be easily seen that from Equation 1 that

$$a = \left(\frac{2m}{2n + 1}\right)^{\frac{2}{3}} (1 - \mu)^{\frac{1}{3}} \quad , \tag{5}$$

where μ is the moon's fraction of the mass of the earth-moon system and is equal to 0.01215. Thus, Equation 5 determines the semimajor axis a of the required orbit of the vehicle in terms of two integers m and n. Once a is determined, e can be obtained from Equation 2 provided that α is given.

The computed values of a for different integers m and n are given in Table 1. It is apparent that large values of m are not useful because it takes too long to have an encounter between the vehicle and the moon. Therefore, only the a values for m up to 4 have been tabulated.

Table 1
Values of a for Different Combinations of m and n

n	Value of a			
	$m = 1$	$m = 2$	$m = 3$	$m = 4$
1	0.7600	1.2064	1.5809	1.9150
2	0.5407	0.8583	1.1247	1.3625
3	0.4321	0.6858	0.8986	1.0886
4	0.3654	0.5800	0.7600	0.9207
5	0.3196	0.5073	0.6648	0.8053
6	0.2859	0.4538	0.5947	0.7204
7	0.2599	0.4126	0.5407	0.6550
8	0.2391	0.3796	0.4974	0.6026
9	0.2220	0.3525	0.4619	0.5595
10	0.2077	0.3297	0.4321	0.5234

Actually not all of the entries in Table 1 represent desirable semimajor axes for the orbits of the moon-probing vehicle. This is due to the restriction imposed on the value of α. If α is very near to unity, the space vehicle will be strongly perturbed by the moon or will even collide with its surface, and consequently it would not return to the neighborhood of the earth. On the other hand, if α is considerably different from unity, the vehicle will be too far away from the moon for a successful moon probe. The desired value of α may be tentatively set at a value between 1.08 and 1.20. This is, of course, only for α at the starting point of our successive approximation; the actual value of α corresponding to the final orbit will be different from that of the starting orbit. However, since the effect of the moon on the vehicle is important only in a short time-interval, we should expect that the final α may not be greatly different from the starting value. This expectation is borne out in the next section.

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Now α is further limited by the condition that the orbit of the vehicle should be an ellipse; that is, e must be less than 1. It follows from Equation 2 that

$$\alpha < 2a \quad . \tag{6}$$

If α must be greater than 1.08, those values of $a < 0.54$ can be eliminated immediately. On the other hand, e must be greater than zero, and hence

$$\alpha > a \quad . \tag{7}$$

If α is required to be less than 1.2 so that the vehicle can reach points near the moon, those values of $a > 1.2$ must be excluded.

For the practical considerations, such as the launching of the vehicle and later communications with it, we prefer that $a(1 - e)$ be not too large. Let us suppose that it is necessary to restrict $a(1 - e)$ to be less than γ where γ is a numerical factor of our choice. It follows then that

$$a < \frac{\alpha + \gamma}{2} \quad . \tag{8}$$

If the largest value for α which is still meaningful for a moon probe is taken as 1.2 and if $\gamma = 0.5$, then a must be less than 0.85 according to Equation 8. Therefore, all cases with $a > 0.85$ can be eliminated.

After the values of a which are either greater than 0.85 or smaller than 0.54 have been excluded from our consideration as possible semimajor axes of starting orbits for a moon-probe, Table 1 reduces to Table 2. The present paper will mainly consider the case $m = 1$, $n = 1$.

A PROCEDURE FOR GENERATING THE DESIRED ORBITS

Because of the perturbation by the moon, the starting orbits proposed in the previous section do not represent the true orbit of the vehicle for a moon probe. Since in successive passes the vehicle will be near the moon only when the latter is located on the same portion of its orbit (that is, in the neighborhood of M_1 in Figure 1), the moon may be regarded as in circular motion when its perturbation on the vehicle is treated. This reduces the perturbation calculation to the integration of the differential equations in the restricted three-body

Table 2

Values of $0.85 > a > 0.53$ for Different Combinations of m and n

n	Values of $0.85 > a > 0.53$			
	$m = 1$	$m = 2$	$m = 3$	$m = 4$
1	0.7600			
2	0.5407			
3		0.6858		
4		0.5800	0.7600	
5			0.6648	0.8053
6			0.5947	0.7204
7			0.5407	0.6550
8				0.6026
9				0.5595

problem. The result thus derived does not give exactly the required orbit for the moon probe in the sun-earth-moon system. However, it represents a good approximation to the required orbit. Also, it provides a new starting point for further successive approximation under more realistic conditions for the motions of the moon and the earth around each other and around the sun.

For the time being, then, let us study the orbit of the vehicle within the framework of the restricted three-body problem. Following the usual notation (for example, Reference 4) we use a coordinate system rotating with the moon and with its origin located at the common center of mass of the earth and moon. Also, the separation between the earth and the moon will be chosen as the unit of length. Moreover, the total mass of the system will be taken as unity; therefore, the mass of the earth is $(1 - \mu)$ and that of the moon is $\mu = 0.01215$. In this system of units, the period of the moon around the earth is 2π.

If we now confine the third body, the space vehicle, to the orbital plane of the moon around the earth, the equations of its motion assume the form:

$$\frac{d^2x}{dt^2} - 2\frac{dy}{dt} = x - (1-\mu)\frac{x - x_1}{r_1^3} - \mu\frac{x - x_2}{r_2^3} , \tag{9}$$

and

$$\frac{d^2y}{dt^2} + 2\frac{dx}{dt} = y - (1-\mu)\frac{y}{r_1^3} - \mu\frac{y}{r_2^3} , \tag{10}$$

where r_1 and r_2 are distances of the third body from the earth and the moon, the latter two being located at $(x_1, 0)$ and $(x_2, 0)$ respectively, and

$$\left.\begin{aligned} x_1 &= -\mu , \\ x_2 &= 1 - \mu . \end{aligned}\right\} \tag{11}$$

It is well known that Equations 9 and 10 admit an integral of the following form:

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$$x^2 + y^2 + \frac{2(1-\mu)}{r_1} + \frac{2\mu}{r_2} - \left(\frac{dx}{dt}\right)^2 - \left(\frac{dy}{dt}\right)^2 = C \tag{12}$$

where C is the constant of integration.

The moon probe vehicle is supposed to be launched at the perigee (denoted by S_1 in Figure 1) whose coordinates in the present reference system are

$$\left.\begin{aligned} x &= -a(1-e)-\mu \ , \\ y &= 0 \ , \end{aligned}\right\} \tag{13}$$

and whose distance from the earth is $a(1-e)$. The launching velocity necessary to place the vehicle into the starting orbit proposed previously can be computed easily. In the rotating system of reference, the two components are given approximately by

$$\left.\begin{aligned} \frac{dx}{dt} &= 0 \ , \\ \frac{dy}{dt} &= \mp \left[\frac{1+e}{a(1-e)}\right]^{\frac{1}{2}} + a(1-e) \ . \end{aligned}\right\} \tag{14}$$

The minus sign in the equation for dy/dt denotes the ejection which will lead to orbits rotating in the same sense as the moon around the earth (direct orbits), while the plus sign leads to orbits rotating in the opposite sense as the moon (retrograde orbits). It is therefore obvious that the moon's perturbing effect on the vehicle is greater in the first case than in the second because when the vehicle and the moon are revolving in the same sense their encounter will last longer than when they are revolving in the opposite sense. Therefore, we would expect that it would be easier to find desired orbits of retrograde motion than to find those of direct motion. Indeed, as will be seen immediately, the orbits revolving in the same sense are unstable.

The magnitude of the launching velocity in the stationary frame of reference is roughly

$$V = \left[\frac{1+e}{a(1-e)}\right]^{\frac{1}{2}} \tag{15}$$

which is, of course, more important than $(dy/dt)_{S_1}$, that is, dy/dt at point S_1 (see Figure 1), for practical considerations.

Equations 9 and 10 can be integrated with the four initial conditions given by Equations 13 and 14; thus, the required orbits are found by successive approximation. The integrations were carried out on the IBM 7090 digital computer at Goddard Space Flight Center. The Range-Kutta method was used, with $\Delta t = 0.01$.

For the case $m = 1$ and $n = 1$ in which a is fixed at 0.7600, the equations were first integrated under the initial conditions given by Equations 13 and 14 for different values of α, or equivalently for different values of e according to Equation 2. Even with this first trial run it was found that, for the positive initial velocity – that is, with the plus sign in the second of Equations 14 – the vehicle will return in some cases of α very nearly to its initial conditions after a duration of about two sidereal months. This shows that these orbits are near to the required one that will encircle both the earth and the moon for a long period of time. For a detailed study we take $\alpha = 1.14$ which is in the middle of the range of interest 1.08 to 1.20. In this case, the initial conditions at point S_1 follow directly from Equations 13 and 14 and are numerically equal to

$$\left.\begin{aligned} x &= -0.39215 \,, \\ y &= 0 \,, \\ \frac{dx}{dt} &= 0 \,, \\ \frac{dy}{dt} &= 2.3670 \,. \end{aligned}\right\} \qquad (16)$$

It could then be argued that the desired orbits which must be closed ones in the present frame of reference should have the property that at their closest approach both to the moon and to the earth

$$\left.\begin{aligned} y &= 0 \,, \\ \frac{dx}{dt} &= 0 \,. \end{aligned}\right\} \qquad (17)$$

These two conditions provided a basis for determination of the correct value, if any, of $(dy/dt)_{S_1}$ in order to generate a closed orbit. Thus, we obtain the correct value of $(dy/dt)_{S_1} = 2.35165$. A final integration with this value as the initial condition was then performed up to $t = 240$. During this time interval of nearly 3 years (38.20 sidereal months), 19 encounters with the moon were obtained. The orbit repeats itself after each encounter. This indicates that the orbit (Figure 2) is a closed one for all practical purposes.

It should be mentioned in passing that it took the IBM 7090 computer about half an hour to compute, with double precision, the path of the third body from $t = 0$ to $t = 240$ with $\Delta t = 0.01$.

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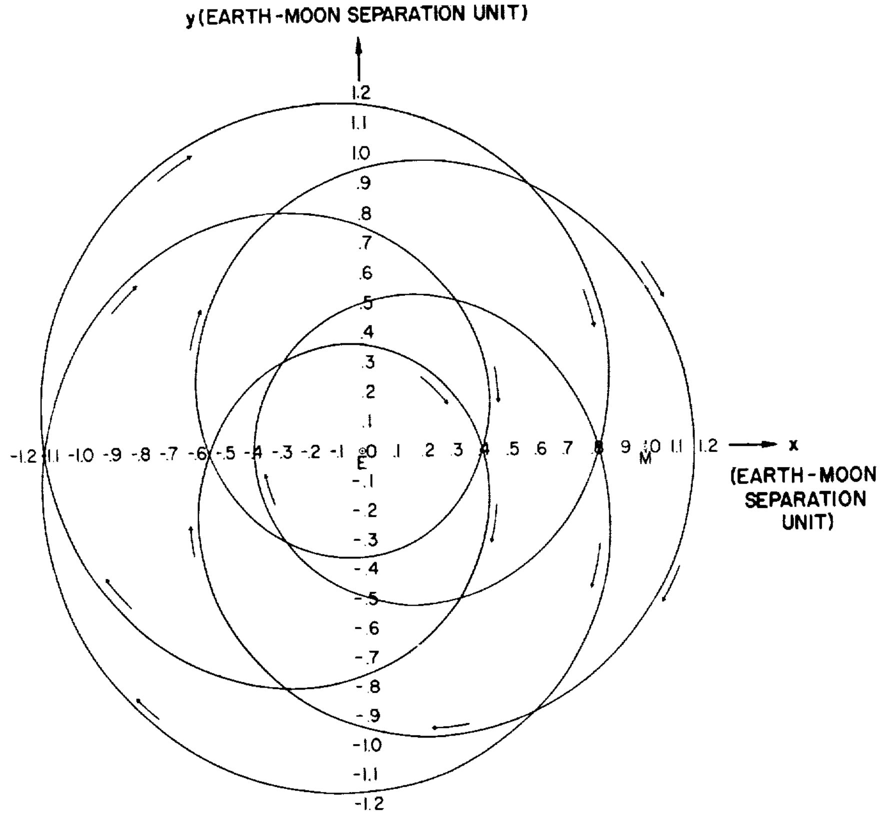

Figure 2—A retrograde orbit which makes periodic encounters with the moon. The orbit is drawn in the frame of reference rotating with the earth E and the moon M.

It should be pointed out here that the final α , as is seen from Figure 2, is about 1.16, which is much larger than the initial value of 1.14. Thus, if it is desirable for the third body to be closer to the moon during its passage there, the computation should be started with a smaller initial value of α.

The desired orbit corresponding to negative ejection, i.e., with the minus sign in the second of Equations 14, is not so easy to generate. In the case of α = 1.14, the first trial under the initial conditions at point S_1,

$$\left.\begin{aligned} x &= -0.39215 \, , \\ y &= 0 \, , \\ \frac{dx}{dt} &= 0 \, , \\ \frac{dy}{dt} &= -1.6070 \, , \end{aligned}\right\} \qquad (18)$$

which follow directly from Equations 13 and 14, leads to an orbit which does not return to the initial space and velocity coordinates. After trials it was found that the correct value of $(dy/dt)_{s_1}$ should be around -1.61025. A further integration with this value was performed up to $t = 60$, but the result is not quite satisfactory. After three passages around the moon, which are identical within the accuracy of the plotted figure, the third body deviates greatly at the fourth passage and never reaches the other side of the moon again before $t = 60$. The orbit before the fourth encounter with the moon is illustrated in Figure 3.

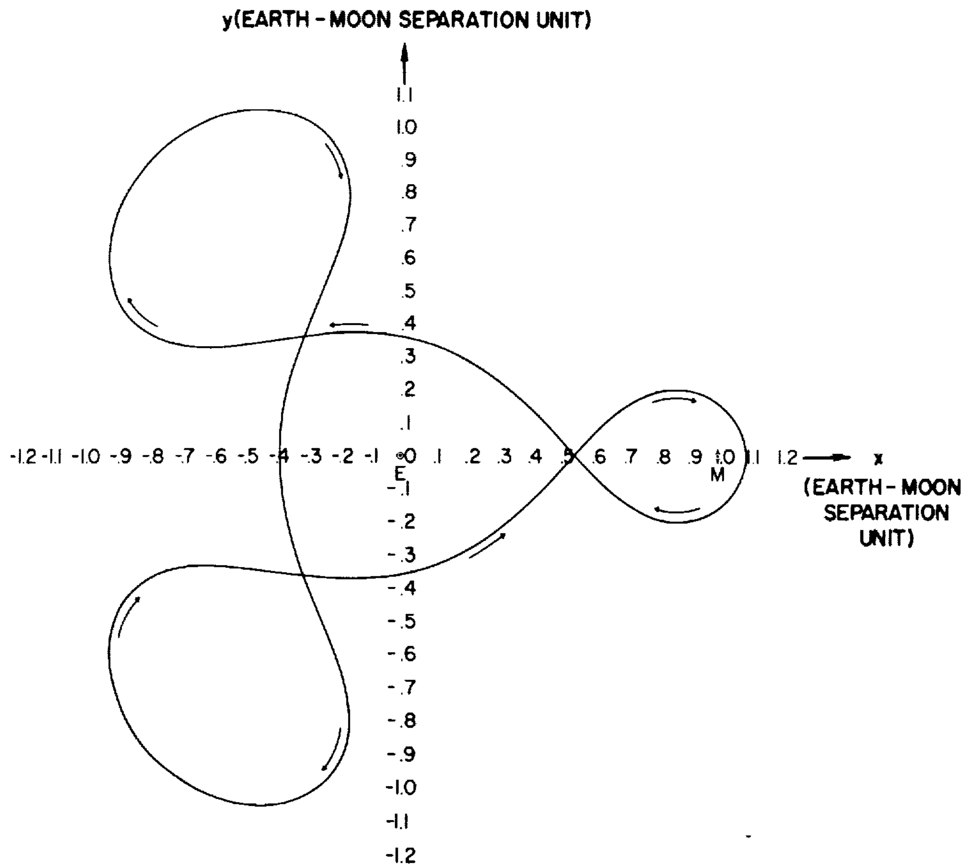

Figure 3—A direct orbit which makes periodic encounters with the moon. The orbit is drawn in the frame of reference rotating with the earth E and the moon M.

It can be seen from Figure 3 that the final a is about 1.07 which, contrary to the case of positive ejection, is much smaller than the initial value of 1.14. The strong perturbation by the moon at the time of close encounter is the reason why a closed orbit is so difficult to obtain in the present case. However, the orbit has the advantage that it gives the probing vehicle a longer time to look at the other side of the moon than the retrograde orbit can provide.

In Figure 4 we have converted the closed orbit in Figure 3 into the actual path in the stationary coordinate system (denoted by ξ and η). The heavy line represents the circular orbit of the fictitious moon. The complete path of the third body covered in time from 0 to 60 has not been drawn because it would make the figure too cluttered to be clearly discerned. Only those portions of the path are drawn that indicate critically the intrinsic behavior of the orbit. Thus, the third body is launched at $t = 0$ at the point marked by 0 when the fictitious moon is at the point marked also by 0 on the circular orbit. The third body at first moves on nearly elliptical orbit because the effect of the moon is small. After about one and a half revolutions during which the fictitious moon has traveled a little less than a complete revolution around the earth, the third body and the moon have a close encounter at about $t = 6$. In the figure, the positions of the moon and the third body at $t = 5.68$ and $t = 6.04$ have been indicated; obviously the closest encounter occurs between these times.

The encounter perturbs the third body so strongly that the line of apsides of its orbit is rotated an appreciable angle as is clearly seen in the figure. The third body now revolves on a new orbit. Only a little more than one revolution of the new orbit is shown in the figure. Actually there are a little less than three complete revolutions before the third body has another close encounter with the moon. The second encounter occurs at a time between $t = 17.28$ and $t = 17.68$; the positions of both the moon and the third body at these times are again marked in the figure. The second encounter again perturbs the third body into a new orbit. This process of shifting the line of apsides repeats itself in a time interval of a little less than three periods of the third body (or a little less than two sideral months).

If the successive encounters only make the line of apsides rotate without affecting other elements of the third body's orbit, the orbit will be a stable one; that is, the third body will revolve around the moon as well as around the earth for a long period of time. Actually the perturbation does cause the changes in other orbital elements. These changes slowly destroy the synchronization of the motions of the moon and the third body. Because the moon and the third body are revolving around the earth in the same sense, a slight shift in phase of the encounter eventually brings the two bodies closer together during the encounter. Therefore, the perturbation increases rapidly and destroys completely the synchronization. For instance, the semimajor axis, and consequently the period, of the third body's orbit is greatly increased after the fourth encounter (Figure 4). The third body no longer revolves around both the earth and the moon thereafter.

Another difficulty of this direct orbit for practical use is the large angle with which the orbit rotates after each of the first three encounters. Since the moon's orbit is not actually circular but elliptical, the change in the separation between the moon and the earth would destroy the regularity even at the first encounter.

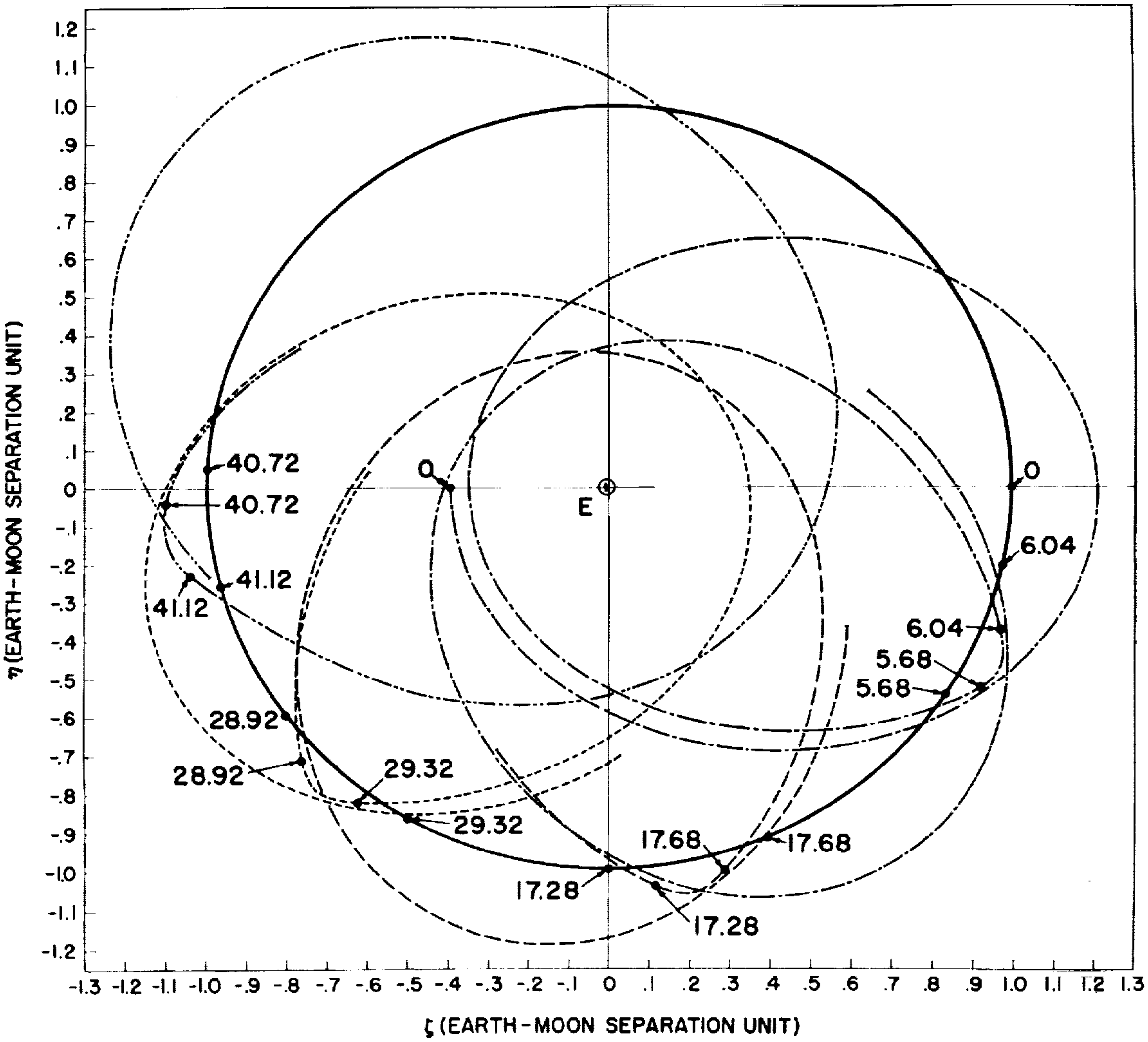

Figure 4—The direct orbit of Figure 3 seen in the stationary frame of reference. The heavy circle represents the moon's orbit. Numbers denote times of passage of the moon and of the third body at various points on their respective orbits near each close encounter. For example, the first close encounter occurs between $t = 5.68$ and $t = 6.04$.

The retrograde orbit shown in Figure 2 is difficult to draw in the stationary frame of reference because the orbits of the third body in different revolutions around the earth are so crowded that they cannot be discerned when plotted together in one single diagram. Indeed, the nature of perturbation on the third body in the retrograde orbit is quite different from that in the direct orbit. In the latter case, the moon and the third body have a close encounter of long duration nearly every three revolutions of the third body, while for the rest of the time they are quite far apart. Consequently, the modification of the orbit as a result of the lunar perturbation occurs suddenly during the encounter – as can be seen in Figure 4 – but is not appreciable in other times. With the retrograde orbit, on the other

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hand, encounters are more frequent – though less drastic because of shorter duration. Hence, the modification of the orbit occurs gradually. In other words, the orbit is drifting slowly in contradistinction to the sudden shift in the case of the direct orbit. Moreover, the particular orbit that has been considered here for the retrograde motion has a much more distant encounter with the moon than does the orbit of direct motion that has first been discussed. Thus, the perturbation is smaller accordingly, and the third body in the present case moves around the earth in nearly the same region of space again and again. For this reason it would be very confusing if the path corresponding to Figure 2 were plotted in the stationary coordinates. To illustrate the general behavior of the retrograde orbit, the portion of the path near the moon is shown separately at different encounters is shown in Figure 5. Just as in Figure 4, the positions of the moon and the third body at labeled times are marked on the orbits respectively. Here the regularity of the encounters can be seen most clearly.

In both direct and retrograde orbits, the line of apsides retreats as a result of encounters with the moon. Consequently, the time interval between two consecutive encounters is slightly more than two sidereal months in the case of the retrograde orbits and less than

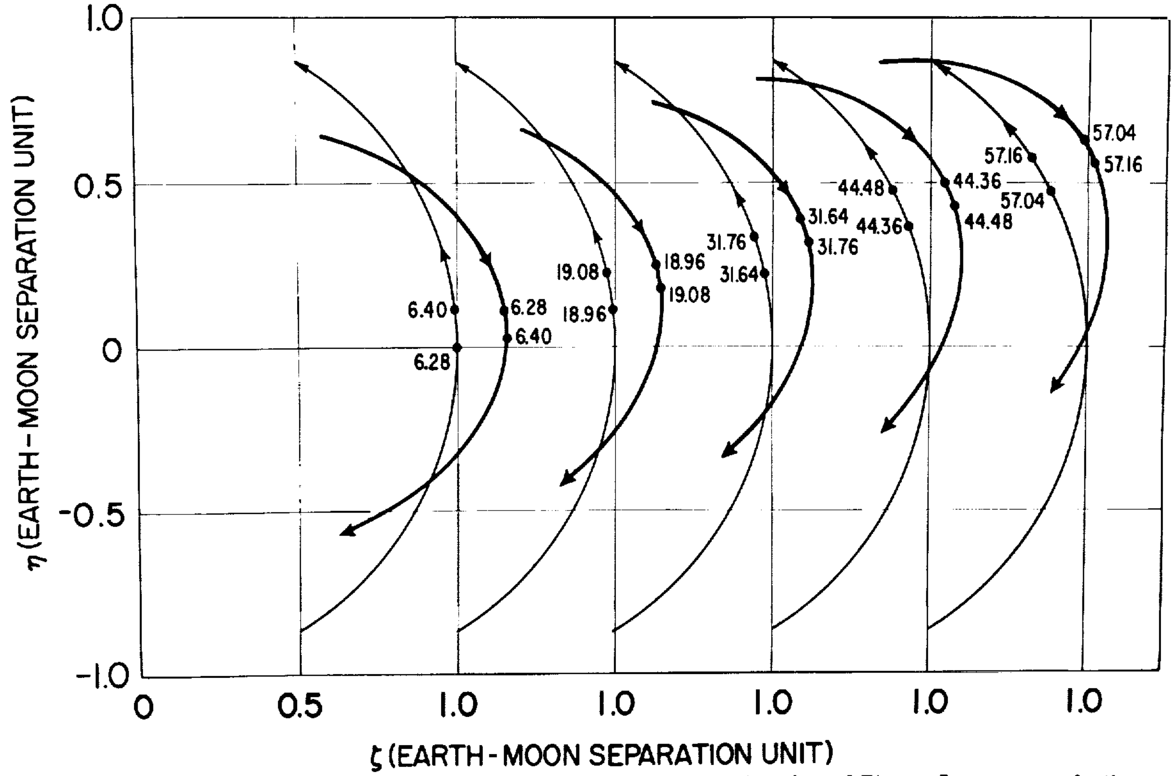

Figure 5–The first five encounters with the moon of the retrograde orbit of Figure 2, seen now in the stationary frame of reference. Plotted here are the motion of the moon (upwards) and that of the third body (downwards). Numbers have the same significance as in Figure 4. The regularity of encounters can be seen clearly, the period deviating from two sidereal months because of the motion of the line of apsides of the third body's orbit.

two sidereal months in the case of direct orbits. As has been mentioned before, the shift in the line of apsides is not a favorable feature if it is desired to apply the result under the present approximation of circular motion to the actual earth-moon system of elliptical motion ($e = 0.05490$).

STABILITY

The possibility of finding the desired orbits in the actual system of the earth and the moon depends ultimately upon the tolerance in the initial conditions being such that the orbits obtained under the approximations of the restricted three-body problem will not be destroyed immediately. In order to examine this tolerance, six more cases were integrated for positive ejection with $(dy/dt)_{s_1}$ to deviate from the correct value of 2.35165 by ±0.05, ±0.10, and ±0.15 percent, but with no change in other initial conditions. For each case the equations were integrated up to $t = 60$. Figure 6 illustrates the behavior of the resulting orbits in the stationary frame of reference. Just as in Figure 5, only the portion of the orbit during the encounter with the moon is drawn. In each diagram the percentage deviation from the correct value of $(dy/dt)_{s_1}$ is marked at the upper left corner. Five encounters are shown in each case. The dots mark the position of the moon and the third body at the labeled times during encounters.

From Figure 6 it can be noticed that synchronization of the motions of the moon and third body is completely destroyed in the case of -0.15 percent after the third encounter, which takes place in the wrong side of the moon, while in other cases the regularity is maintained up to $t = 60$. These orbits, which should be compared with the ideal case illustrated in Figure 5, undergo oscillations in the distance of the encounter. This appears to indicate the stability of the orbit under a small change in initial conditions. We are encouraged by this property to predict that an orbit encircling both the earth and the moon for a period of a few encounters is obtainable.

Similarly, the equations for six more cases were integrated in connection with negative ejection. Their initial conditions follow Equations 18 except with $(dy/dt)_{s_1}$ being ±0.005, ±0.010, and ±0.015 percent from the correct value of -1.61025. Note that the percentage changes are only one-tenth of those considered for the retrograde orbit. The results are shown in Figure 7. As with the case of Figure 6, only that portion of the path which encounters the moon is plotted. Similarly, the times and the positions of both the moon and the third body during each encounter are marked in the diagram. All orbits in the figure make only two encounters with the moon before they are perturbed out of synchronization. This shows that the orbit is not stable under a slight change in the initial conditions. Actually this fact can be expected from the behavior of the orbit shown in Figure 4, in which a slight phase shift in synchronizations at the fourth encounter drastically changes the nature of the orbit so that synchronization is completely lost thereafter.

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Although the distance of encounter in our example (Figure 7) is too short to derive a general conclusion, we expect that this instability will not disappear even in those synchronizing orbits which make more distant encounters with the moon. It can be easily seen intuitively that when the moon and the third body revolve around the earth in the same plane and in the same sense, a slight shift in phase of synchronization in both directions will

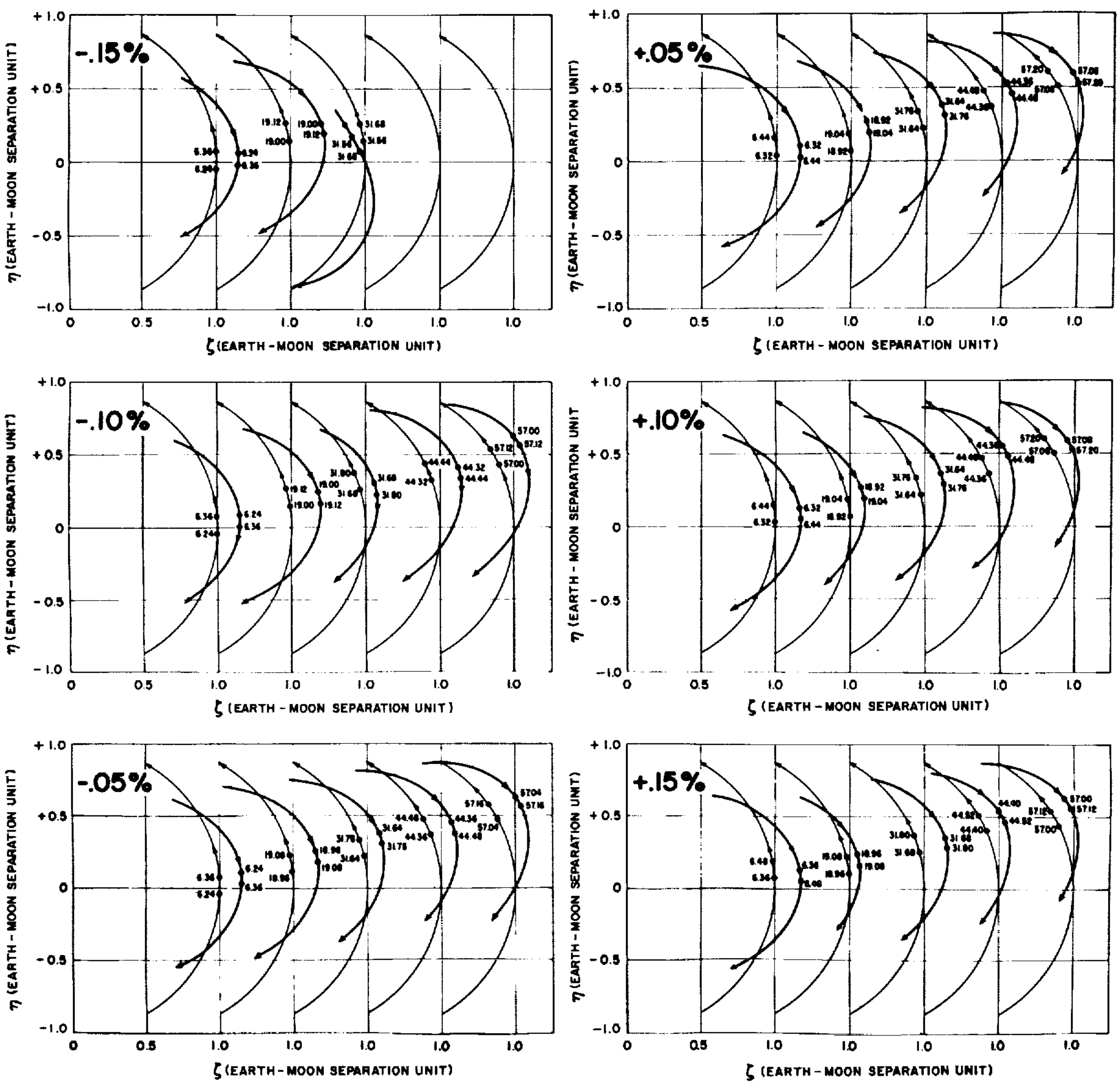

Figure 6—The effect of a small change in the initial conditions on the stability of the retrograde orbit. The percentage deviation of the launching velocity from the correct value is marked at the upper left corner in each diagram which is, in all other respects similar to Figure 5. Except for the case of -0.15 percent, all show stability of the encounters.

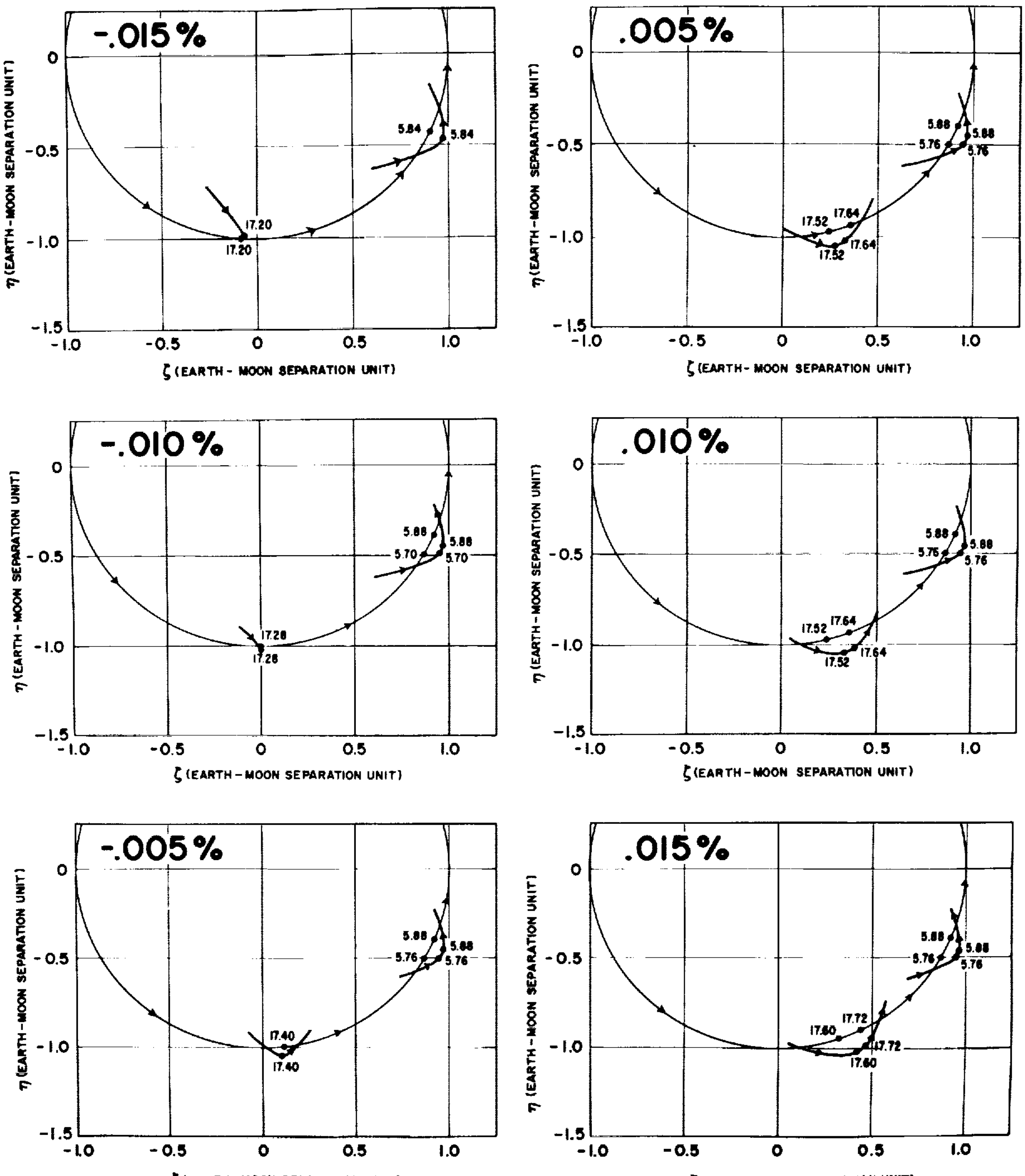

Figure 7—The effect of a small change in the initial conditions on the stability of the direct orbit. Even with such a small deviation in launching velocity (1/10 the deviation of Figure 6), no more than two encounters can be obtained; this shows the instability of the orbit. From both Figures 6 and 7, the deviation in the launching velocity appears to be less serious on the positive side than on the negative side; this fact has practical significance.

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result a very close encounter and thus necessarily modify the orbit of the third body in a drastic way. Consequently, if orbits encircling both the earth and the moon are required, they should be looked for among the retrograde orbits.

CONCLUDING REMARKS

Although we have studied in detail only the case in which the initial value of α is 1.14, it is obvious that the same procedure can be used to derive other synchronizing orbits corresponding to other initial values of α. In this way a one-parameter family of orbits with the desired property of encircling both of two finite bodies can be derived for positive ejection as well as for negative ejection. For other pairs of values of m and n, other families of orbits with the same property can be similarly generated. So there are many families of desired orbits encircling both the earth and the moon under the approximation of the restricted three-body problem. However, not all of them are qualified for actual use once the eccentricity of the moon's orbit is taken into account. We would suggest that a few good ones be chosen from these families of orbits, as the starting condition for further successive approximation with both the moon's orbital eccentricity and the presence of the sun taken into account.

Because of practical considerations, this report has dealt with the encounter of the moon with the third body near the apogee of the latter's orbit. However, we should add that retrograde orbits which make the encounter occur at the third body's perigee are expected to be more stable than these, simply from the fact that the perigee encounter lasts a shorter time. This possibility should be considered if we require an orbit that will encircle both the earth and the moon for a few decades or more.

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